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When spread bites fast – Volatility and wide bid-ask spread in a mixed high-frequency and low-frequency environment

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A B S T R A C T

This research focuses on the impact High-Frequency Trading has on price volatility when bid-ask spread is wide. The theoretical part introduces a set of equations and presents an Agent Based Model implemented via a computer-based simulation. The wide spread leads to the appearance of unusual phenomena caused by the relative speed difference between the fast and slow traders. The latter agents tend to quote limit orders that look irrational, as they are distant more than one tick from the top-of-book. The same relative speed difference causes slow traders to post market orders that execute at price worse than originally intended. Both these abnormal orders tend to increase local volatility. Other results found by the simulation are an increase in global volatility (computed both as the difference of maximum less minimum price and as standard deviation of price distribution) and in volatility at sub-second timescales. These occurrences penalise slower traders and affect market stability. All the results are consistent both under quiet and stressed market conditions. The results found are then compared with audit trail data to verify the soundness of theory against practice.

1. Introduction

Financial stability has long been a major concern for market participants, surveillance authorities and policy makers alike, the mantra of the community being the orderly functioning of the system. It must be said that the goal is usually being hit - with exceptions, though. Financial stability differs from country to country, because of different viewpoints often driven by past history of that specific country; yet most observers agree to identify in a bunch of features the core of financial stability: price volatility and a few others, as liquidity, price discovery and bid-ask spread, at some extent related to it. Controlling volatility is crucial for relaying the impression of financial market as stable and trustworthy institutions, where risk is well managed and ordinary people can confidently invest their savings, sure that the bet is within their limits of risk tolerance. Needless to say, this has not always been the case in financial history. Bubbles and crashes are relatively common occurrences (Sornette, 2003; Johansen and Sornette, 2010 among others) and even the most casual investors are well aware of such possibilities. On the other side, a certain degree of volatility is the precondition for the very existence of a financial market. A dead still market would not appeal anyone because of complete lack of profit opportunities. In order to have winners there must be price movements - and losers. Experts as well as amateurs seem to put a lot of faith in the reassuring shape of the Gaussian bell when forecasting the price movements in the foreseeable future. Within a certain range of standard deviations, volatility is a welcome feature of the markets but when it trespasses the borders of such range (borders that vary a lot with personal circumstances, risk aversion and tastes) the fat-tail nightmares materialise and volatility is blamed as the source of all evils. If the cause of volatility has never been a thoroughly understood matter –and the long list of

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financial crises demonstrates that– the relatively recent introduction of High-Frequency Trading (HFT) as a widely adopted strategy has further exacerbated the issue (Aldridge and Krawciw, 2015) and according to Serbera and Paumard (2016) HFT participation does not tend to diminish, despite reducing profitability. May 6, 2010 shall be remembered by market insiders as the day of the Flash Crash, a day that recorded an unheard of spike of volatility, when the US markets very quickly lost, and almost as rapidly recovered, one full trillion dollars (CFTC-SEC, 2010). Aside the Flash Crash, there seems to be a large number of minor but similar events, called mini flash crashes, that according to some authors count as several on each trading day (Johnson et al., 2013) and would be directly related to ultra-fast trading strategies. Therefore, thoroughly studying and fully understanding the causal relationship, if any, between HFT and volatility has in recent years become one of the most compelling tasks of disciplines as market microstructure, risk management and financial economics as well as policy-making. This research contributes to such study by focussing on market conditions characterised in particular by wide bid-ask spread, not an infrequent occurrence. The purpose of this article is to verify whether HFT and wide bid-ask spread caused by exogenous factors can create the conditions for financial instability, namely Naïve Orders and Bad Deals. According to several authors (Brogaard, 2010; Jarrow and Protter, 2012; Aitken et al., 2012; Menkveld, 2013; Myers and Gerig, 2014), HF traders often provide liquidity to the books, so reducing spread. Yet, this behaviour does not seem consistent under both quiet and stressed conditions (Kirilenko et al., 2017) and some authors also characterise HFT market making as ‘phantom’ liquidity provision (AFM, 2016; Arnoldi, 2016; Menkveld, 2016), when the same securities are quoted onto several venues and immediately cancelled as soon as one execution occurs on one of them.

In the rest of this article, section 2 reviews the recent literature, section 3 describes the theoretical model whereas section 4 describes the Agent-Based Model used for the simulation. Section 5 analyses the results of the simulation, section 6 presents audit trail data and section 7 compares and discusses the results of the simulation and audit trail data. Section 8 concludes.

2. Literature review

High-Frequency Trading has dictated the rules of the game by setting up an environment in which its timing became the timing of the market as a whole. If volatility leans itself to be modelled as a random variable, according to Gatheral (2006) stochastic volatility processes can be regarded as a form of Brownian motion dependent from a random clock. Such time randomness can only be exacerbated by the sub-second timescale at which volatility processes currently operate. When the time of the markets was gauged by human perception, the volatility process could be regarded as continuous. But at sub-second scale this is hardly true. The ‘sub-second scale’ is no longer meant to be the millisecond (10^{-3} second) timeframe but rather microsecond (10^{-6} sec) and counting toward the nanosecond (10^{-9} sec) threshold. At these timescales peaks and troughs appear where none seemed to exist and trading pauses are now the norm rather than exception. As Anderson et al. (2015) put it, "although episodes of heightened volatility and short-term illiquidity are not necessarily in themselves threats to financial stability, they could become so if they were to persist, amplify or spill over" (Anderson et al., 2015. p.4). The common goal of exchanges, non-speculative as well as institutional investors, and policy makers is to achieve moderate volatility at all times.

2.1. Volatility and high-frequency trading

A model of a market in which operate momentum traders as well as informed and noise traders has been developed by Gsell (2008), leading to the conclusion that fast trading significantly reduces volatility. Another model of a continuous double auction market, implemented as an Agent-Based simulation, has been produced by Myers and Gerig (2014) and also used to demonstrate a decline in volatility. Similar results found Hasbrouck and Saar (2013), who observe that the more rises low-latency activity and the more it lowers volatility in the short-term. The thirty most actively traded securities on the NASDAQ Stockholm exchange were analysed in Hagströmer and Nordén (2013), finding that HFT acts as a mitigating factor of price volatility. The main futures markets were analysed by Bollen and Whaley (2015), yielding similar results. A sample of the price at closing for the 200 most actively traded stocks in the S&P500 basket during the 1985–2012 period allowed Kelejian and Mukerji (2016) to conclude that HFT reduces correlation between fundamentals and volatility. Yet, not all researchers are so clear-cut about the beneficial effects of HFT on volatility. Zervoudakis et al. (2012) admitted of being unable to formulate unambiguous causal relationship between HFT and volatility. Also Brogaard (2010) does not find strong evidence of HFT reducing volatility and similar failure to find direct evidence of negative correlation between HFT and volatility is reported by Zingrand et al. (2012), who argue that, in some cases, HFT can even increase volatility. The same occurrence is observed by Abrol et al. (2016), who find that self-reinforcing feedback loops could exacerbate volatility, generate shocks and increase risk at systemic level. Jarrow and Protter (2012) investigate how HFT exploits arbitrage opportunities and found that as many High-Frequency (HF) traders adopt similar strategies, price movements are amplified. Crowding effect in more general terms, and not limited to arbitrage only, is also interpreted by Brogaard (2010) as a confirmation that similar HFT strategies have the potential of magnify market movements and Chaboud et al. (2014) find statistically significant correlation between strategies adopted by HF traders, which would increase volatility. Common HFT strategies have also been found by Kirilenko et al. (2017), who studied the short time window after prices displaying clear directional movements. Also bold are the results stated by Zhang (2010), who finds a correlation between HFT and volatility by analysing exogenous shocks of NYSE autoquote. Aldridge and Krawciw (2015) find a positive correlation between aggressive HFT activity and volatility; yet, they refrain from stating any causal relationship: "[i]t is not immediately clear […] whether aggressive HFTs seek out high volatility, whether aggressive HFT participation induces higher volatility in stocks, or both”. Overall, the academic community displays diversity of opinions about the impact HFT would have on volatility. This paper addresses the impact of HFT on volatility under specified conditions.

2.2. Volatility and bid-ask spread

An Agent-Based Model (ABM) has been developed by Vuorenmaa and Wang (2014) who state an impact of tick size, a determinant of bid-ask spread, on volatility. Their research finds that an increase of tick size leads to a decrease in the number of available price levels in the order-book close to the mid-quote, therefore increasing liquidity at each level, and eventually reducing volatility. There is an opposite effect, however. As an increase of tick size also increases the bid-ask spread, noise trading impact on prices is also increased and this amplifies volatility. Dependence of volatility on size of bid-ask spread has not received much attention by the existing literature; this research attempts to fill the gap.

2.3. Volatility at sub-second timescale

Extreme volatility is a topic that has been put at the forefront by the entrance of HFT to the financial markets. Several sources talk about the so-called mini flash crashes, events occurring at sub-second scale, affecting only one or very few securities and therefore much less noticeable than the May 6, 2010 global event. Analysing the market data supplied by analysis firm Nanex, Durden (2013) identifies thousands of mini flash crashes over the last few years and similarly Foresight (2012) recognises that "[t]here has been a variety of other, smaller illiquidity events in the markets since the Flash Crash" (Foresight, 2012. p.57). Even more enlightening are the findings of Johnson et al. (2013) that grouped short-time extreme-price events by duration in hundredths of milliseconds time windows. The results lead to the conclusion that over the six-year period 2006–2011 there were 18,000+ extreme events on the main US exchanges. This kind of 'local volatility' becomes important when a large proportion of trading activity occurs at sub-second timescales and it shall be discussed further in this article.

2.4. HFT exacerbating effect on volatility

As seen before, several studies find that HFT tends to mitigate volatility at normal times but some recognise a different behaviour and a different effect under stress. This seems confirmed by the findings of Golub et al. (2012), a research analysing mini-flash crashes on 2006–2011. Their results suggest an amplifying effect of HFT on volatility during such critical times, an outcome confirmed by Fry and Serbera (2017), particularly for highly capitalised stocks. Kirilenklo et al. (2017) also recognise that, although HFT may not be the prime responsible of crashes, its action might exacerbate existing volatility. Differences between market behaviour in normal times and under stress is also a topic this research deals with. The wide difference in viewpoints about the relationship between HFT and volatility leads to conclusion that research on this field is not over. This paper contributes to the debate by producing a mathematical model and a computer simulation of a market in which both HF traders and Low-Frequency (LF) traders interact. Then it verifies whether the scenario suggested by the results of the theory matches the evidence resulting by analysis of audit trail data.

3. Description of the theoretical model

Let P denote the price of a risky asset with supply functions X. A trading cycle is made up of three dates for each trading cycle. At date 1 liquidity providers start evaluating signal Θ_m (the suffix *m* indicating market makers) and at date 2 they quote liquidity X_m at price P_m. At the same time aggressive traders (market takers) learn both values and start evaluating their own price P_t, using signal Θ_t, and at date 3 they post market orders.

The relevant equation at date 1 is:

$$\Theta_{mn} = \Theta(A_m, X, P, \tau), \tag{1}$$

where n is the cycle identifier, A_m is the risk aversion of liquidity provider m, X is the liquidity on the book, P is the book price and τ is time. Both X and P enter the equation as do their first partial derivatives, $\partial X/\partial\tau$ and $\partial P/\partial\tau$, to take into account their dynamic behaviour. Moreover, as soon as P_m gets published, aggressive traders start evaluating it, together with other signals, performing an operation that also starts at date 2. Then, at date 2 the relevant system of equations is:

$$X_{mn} = X\ (\Theta_m, P_{mn}), \tag{2}$$

$$P_{mn} = P\ (\Theta_m, X_n), \tag{3}$$

$$\Theta_{tn} = \Theta(A_t, X, P, \tau), \tag{4}$$

where A_t is the risk aversion of liquidity consumers.

When evaluation is completed, liquidity consumers post aggressive orders and market clears immediately. This obviously only happens iff $P_m \geq P_t$ or $P_m \leq P_t$ (according to whether the trade occurs at the bid or ask, respectively). The relevant equation at date 3 is:

$$P_{tn} = P(\Theta_{tn}, X_n). \tag{5}$$

In an environment operated by HF traders, the three dates happen to be very close to each other, in the scale of milli- or even micro-seconds. A Low-Frequency trader, whether passive or aggressive, may well 'miss' one or more cycles because of its latency λ, for example due to longer signal evaluation time or delay in network connection. Thus, depending on liquidity X and on latency λ, the price function may vary considerably, especially if X is relatively small and/or λ is relatively large. If the price function varies between cycle N and cycle N + n (with n > 0), LF traders may act at cycle N + n with respect to signal $\Theta_{i,N}$ ($i \in \{m, t\}$). In other words, LF traders quote a price misaligned with other quotes in the book or post an order 'at best' that executes at a price different from the one used for evaluating the signal function Θ at cycle N. This may have four consequences, according to which direction the market moved.

1) Although now it is cycle N + n, liquidity provider M uses signal $\Theta_{M,N}$ to quote liquidity X_N at a price $P_{M,N}$, worse than top-of-book (i.e. lower bid or higher ask). It means that M either quoted a bid while the market moved upwards, or quoted an ask in a downward-moving market. It then loses the opportunity to be competitively placed at the top-of-book with respect to other faster liquidity providers.
2) Liquidity provider M quotes liquidity $X_{M,N}$ at a price $P_{M,N}$ more competitive than other traders because it quoted a bid in a downward-moving market, or quoted an ask while the market moved upwards. This is called a "Naïve Order", that is, a limit order sitting (unaware) deep into enemy territory and isolated from its own peers, ready to be picked off by the quickest aggressive predator that evaluates $P_{M,N}$ more advantageous than $P_{t,N+n}$.
3) According to signal ΘT,N, although now it is cycle N + n, a liquidity consumer T posts a buy order at price $P_{T,N}$ lower than lowest ask, or a sell order at price $P_{T,N}$ higher than highest bid. Yet, since this is a market order, it would execute 'at best', even if the best price turns out to be worse than the one originally intended by the LF trader. This is called a "Bad Deal".
4) A liquidity consumer T posts a buy order at price $P_{T,N}$ higher than lowest ask, or a sell order at price $P_{T,N}$ lower than highest bid. In either case exchange rules would force execution of the order at market price P_{N+n}.

Case 4 is the only one in which the delay would favour the slow trader. All other cases are more or less harmful to her.

4. Description of the simulation

As seen in section 2, some researchers acknowledge a reduction in volatility as more HF traders enter the game, although this is by no means a unanimous opinion. A major concern against this view comes from the observation of the different latencies experienced by the fast traders and the traditional ones. The High-Frequency traders make use of the latest technology, including multi gigaflops computers, automatic news reading devices, efficient software, Field Programmable Gate Arrays (FPGA) technology, ultra-fast or dedicated networks and co-location facilities. The result is that the same market is operated upon by two categories of actors, each running at greatly different speed, in the order of magnitude of 1000:1 ratio or more. This means that even the simplest operation a fast player performs, may take a comparably very long time when performed by a slow trader. Quote reading is an example of such a simple operation. Given the above-mentioned speed ratio, while a LF trader evaluates a signal, another trader working at high-frequency may evaluate 1000 of them or, more likely, evaluating the signal once and spending the rest of its large time advantage to act upon the price, potentially leaving its slower counterpart with obsolete and sometimes misleading pricing information. There is no guarantee that the price acted upon by a LF trader will still be valid when its order hits the book. If the liquidity does not evaporate in the meantime, it will still be able to trade at the originally intended price, but if liquidity at the price targeted by the LF trader vanishes, the latter's market order will still execute 'at best', even if the 'best' price is now worse than originally intended. It is therefore likely that the volatility experienced by that security unintendedly increases. The schema runs as follows.

1) An ask limit order is executed, quoting price X.

2) An LF market order intending to buy at price X is posted. Yet, this order will suffer a latency compared to High-Frequency orders.

3…t-1) Other orders are being either executed immediately or delayed, according to whether submitted by HF or LF traders, respectively. Under the assumption of scarce liquidity on the ask book, a sequence of several buy market orders might move the price of the security to $X+\Delta x$ (with $\Delta x > 0$).

t) Eventually the order launched at step 2 arrives at the exchange and gets served. The buy market order is executed at the prevailing price, which is now $X+\Delta x$. The trader gets its order executed at a price worse than originally expected and the market experiences unintended volatility.

Surprisingly enough, the literature shows little quantitative research taking this issue into account. In the simulation presented below, the LF delay just described has been implemented by use of a circular queue, where Low-Frequency orders are temporarily parked until a suitable time period has expired, in order to simulate their latency. Then they shall be executed at the prevailing price at that time, with the consequence of moving the market in unpredictable ways. In the subsequent sections a simulation of this scenario shall be described in detail and its outcome analysed with the help of the appropriate statistical tools with the purpose to verify if the situation depicted above can actually occur or it is just mere theory.

4.1. Implementation details

The ABM that implements the simulation has been built with the purpose to be as realistic as possible, and various features have been included with that goal in mind. As suggested by the literature, two cases will be studied: a quiet market environment, with no particular disturbances, used as reference, and a trending one, when the market prices move systematically either up or down. The quiet or trending cases are determined by the criteria used to set the three operating parameters selected in a quasi-random fashion: book type (bid or ask), order type (limit or market), and trader type (High-Frequency or Low-Frequency). In both cases limit orders are given priority when the bid-ask spread is large (in order not to exceed pre-determined spread width) and market order are more likely if spread is thin. The discriminating value has been set to five ticks in order to allow wide, yet not excessive, spread. This way the spread is kept relatively wide, setting the target environment of this research, that aims to study the behaviour of the markets when the spread is relatively wide, allowing for incoming orders to sit at least two ticks above the previous highest bid, or below the previous lowest ask price. Two ticks distance from the previous price are necessary to discriminate a Naïve Order from one that just improves the book by one tick. At the same time the bid-ask spread must not become excessively large, as that is an infrequent occurrence in the real world. The difference between trader types is mostly given by their relative speed. Whereas HF orders get serviced immediately, LF orders are subject to a queueing delay, simulating their latency.

Since the purpose of this research is to evaluate the impact of HFT on volatility, the simulation has been run under several HF-to-LF ratio scenarios: a market with only Low-Frequency traders will show a zero ratio, whereas for the other scenarios the selected percentages are 33%, 50%, 67%, 80%, 90% and 99% HF activity over the total market activity (HF + LF). The results from these simulations are then compared with the reference scenario in which no HF traders take part, in order to verify the impact that HFT has on the market behaviour. This is not new stuff. Yet, in this new experiment a particular scenario has been studied, namely the one in which the bid-ask spread is wide. This is a rather common case under nervous markets: the spread widens up as investors feel so unsecure of the next-future prices that they do not dare quoting a price nearer the other side of the book, because of fear to be picked off by more informed investors. This original scenario allowed to study in detail two issues in particular: (i) the incidence of market orders (in the following referred to as 'Bad Deals', described in previous sections) that on one side execute at a price penalising the operator that posted it, and on the other side, exacerbate volatility without, and against, explicit consent of the same market participant; and (ii) the insurgence of the so-called 'Naïve Orders'. These are limit orders quoted on the middle of the bid-ask spread, far from either top-of-book (that is why a five-tick spread seems appropriate), that might have achieved the same goal of quoting competitively by improving the top-of-book by only one tick rather than more than one. An example is: bid at 100, ask at 105, tick equal to 1. Any bid limit order at 102 or higher, or ask limit order at 103 or lower, are to be considered naïve, as the purpose of sitting on the top of the book could have been achieved by quoting a bid at 101, or an ask at 104, potentially getting a better deal if the order gets executed. Although these orders have been called naïve, the reason for this phenomenon to occur might not be naivety but rather the relative delay of LF traders, that causes them to quote limit orders based on obsolete information. Fig. 1 shows pictorially the different scenarios. Column A displays the initial bid (white) and ask (black) books; columns B and C show competitive orders at 101 on the bid book and at 104 on the ask book, respectively. Columns D and E show still competitive yet Naïve Orders at 102 (bid book) and 103 (ask book). It looks clear that the orders depicted in Fig. 1, panel B and in panel C, are competitive (better than other orders in the book) and rational, whereas the orders shown in panels D and E are not rational; therefore they have been called 'naïve'. All runs of the simulation follow the same standard structure but in each case the selection of the three main parameters (1: book type, 2: order type, and 3: trader type) is made according to the features that are peculiar to the specific case implemented. The testable hypothesis for this simulation is whether by increasing the number of Bad Deals and Naïve Orders the participation of HF traders increases market volatility.

4.2. Simplifying assumptions

A few simplifying assumptions have been made. No trading strategies are implemented, the only exception being the lesser probability of market orders when the bid-ask spread is wide and the higher probability when it is thin, in order to keep the spread wide enough. The other simplifying features implemented in the simulation are about liquidity: (i) every limit order adds one lot of

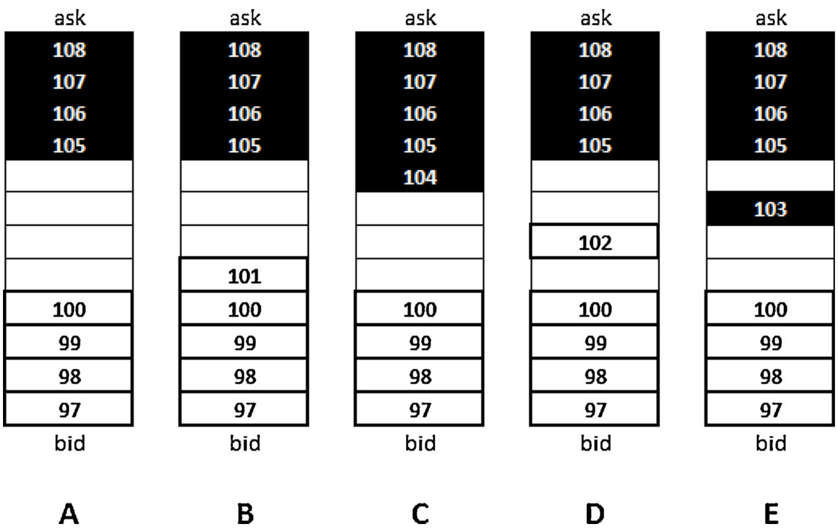

Fig. 1. Rational (panel B and panel C) versus naïve (panel D and panel E) competitive orders. Panel A depicts the initial situation.

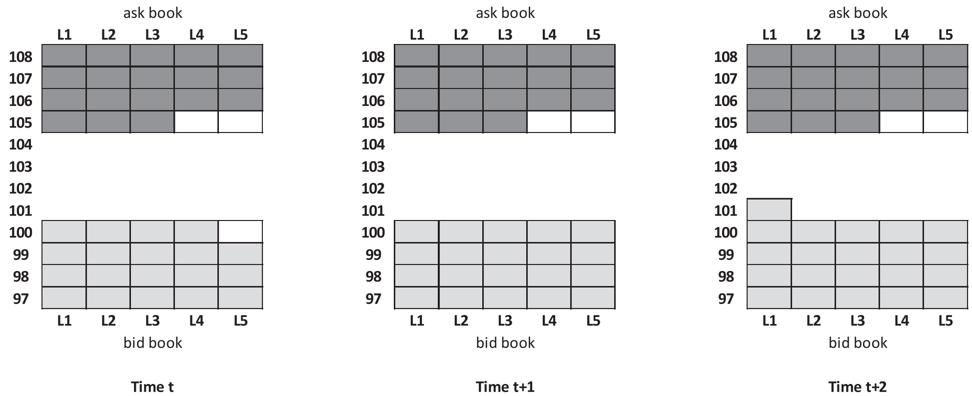

Fig. 2. Liquidity-filling orders. Both bid limit orders are originated at 100 but since at time t + 1 the liquidity reaches its maximum level, the second order is converted into a bid limit order at 101.

liquidity to the book and every market order consumes exactly one lot, and (ii) each price level has a maximum liquidity it can accept (5 lots). The latter feature is useful for simulating the decision to quote an order at the next available price level. The simulation implements this feature, representing the expected behaviour of a rational investor, to mimic the following behaviour. When one price level has reached a certain amount of liquidity, set as the maximum allowed, any further limit order at that price is converted into a limit order at the next price level. The rationale behind this behaviour is that when there are many orders at a certain price level, this mechanism simulates the behaviour of an investor feeling reasonably confident of its and other investors' judgement to dare jumping to the next price level. For example, on the scenario displayed in Fig. 2 the bid book is shown coloured in pale grey and the orders on the ask book are in dark grey. At time t, the bid book shows four liquidity lots filled (L1, L2, L3, and L4) at the top of the book (100), whereas the top of the ask book (105) only contains three lots (L1, L2 and L3). Since the bid-ask spread is 5 ticks (100 bid, 105 ask), only a limit order may appear, as explained earlier. Indeed, two consecutive bid limit orders are being posted. A bid limit order at 100 arrives at time t + 1, filling the last liquidity lot available at that price level (L5). When, at time t + 2, another bid limit order at 100 arrives, it is converted onto a bid limit order at 101 (filling L1), simulating the confidence of the market to quote a higher bid price. At this point, since the spread is four tick wide (105-101), either a limit or a market order may follow. With this mechanism, the bid-ask spread can randomly oscillate between the minimum of one tick and the maximum of five ticks, although it usually hovers around 3–4 ticks.

Table 1
Results of the simulation. The values underneath the simulation results represent the Z-scores. Values not significantly different from LFT-only (i.e. those in the 0% column) at 95% are marked by one asterisk (*) and those at 99% by two asterisks (**).

Panel 'QUIET'	0%	33%	50%	60%	67%	80%	90%	99%
Bad Deals	0.00%	11.48%	10.28%	9.12%	8.13%	5.27%	2.45%	0.25%
		46.23	43.04	40.31	34.94	32.18	29.46	16.04
Naïve Orders	0.00%	12.14%	7.52%	3.72%	1.38%	0.02%	0.00%	0.00%
		23.87	14.56	11.80	9.02	3.09		
Delta Price	0.57	7.36	5.49	3.97	2.73	1.61	1.00	0.85
		18.47	12.91	10.49	8.5	5.05	2.50**	1.67*
Global Volatility	1.26	2.32	2.13	1.92	1.66	1.44	1.42	1.33
		12.25	9.66	7.29	5.04	2.6**	2.41**	0.99*
Local Volatility	0.75	2.18	1.89	1.51	1.16	0.83	0.77	0.75
		25.02	17.21	15.00	16.4	12.78	4.26	1.02*
Panel 'TREND'	**0%**	**33%**	**50%**	**60%**	**67%**	**80%**	**90%**	**99%**
Bad Deals	0.00%	17.63%	16.60%	16.21%	14.01%	8.90%	4.27%	0.37%
		33.93	36.24	56.04	64.03	75.02	50.85	18.83
Naïve Orders	0.00%	8.43%	4.02%	1.70%	0.48%	0.02%	0.00%	0.00%
		15.22	8.65	7.20	5.48	1.77*		
Delta Price	4.60	7.67	9.82	12.20	12.57	10.97	7.99	5.77
		7.03	10.40	15.03	15.89	14.23	7.89	2.83
Global Volatility	2.54	3.13	3.83	4.60	4.73	4.36	3.34	2.76
		4.40	8.62	12.99	13.97	13.07	6.01	1.78*
Local Volatility	0.74	2.32	2.04	1.74	1.41	0.85	0.76	0.73
		21.23	22.00	32.30	37.17	17.55	5.71	−0.63*

5. Results of the simulation

The core of this research consisted in launching the simulation under eight different HF-to-LF ratios, first in a quiet and then in a trending market. The results are shown in Table 1 together with their respective Z-scores. The percentages on the first row of the columns indicate the HF-to-LF ratio. The literature estimates the real ratio around 50%–60% and therefore those are the two most meaningful columns but the others are useful to carry out a 'what-if' analysis or to identify trends. Panel QUIET of the table refers to a quiet market, that follows a random walk pattern (Fama, 1965) whereas Panel TREND simulates a trending market, where prices definitely move either upward or downward. The latter simulation is useful for analysing market behaviour under stressed conditions.

The column headings indicate the %-age of HFT activity over the total (HFT + LFT).

The first column lists the features under observation, namely:

a Bad Deals – the number and percentage of market orders executed by Low-Frequency Traders that resulted in a trade at a price worse than originally intended;
b Naïve Orders – the number and percentage of limit orders quoted more than one tick above (below) the best bid (ask) price;
c Delta Price – the price range displayed during the simulation;
d Global Volatility – the standard deviation of the prices at which a trade has been executed;
e Local Volatility – the difference between two consecutive quoted prices. This number supplies an indication of how volatile is the price at sub-second scale rather than over the time window of the whole simulation.

All the results considered for a HF-to-LF ratio greater than zero are compared with those obtained with a ratio equal to zero and Z-scores are reported below such values. The purpose is to analyse the impact on the various features in presence of HF traffic with respect to the base case, represented by LF traffic only. Therefore, all the hypotheses are tested for equality to the value of the same feature contained in the first column, corresponding to a HF-to-LF ratio equal to 0 (i.e. LF traffic only).

5.1. Bad deals

The percentage of Bad Deals over all trading is significantly greater than zero for all HF-to-LF ratios. It spikes high as soon as some HFT activity hits the market although the percentage of Bad Deals decreases as HFT activity rises. This phenomenon is noticeable both during quiet times and even more under stressed conditions (trending market). As long as there is some LFT activity, its delay condemns it to execute some trades at a price worse than originally intended. A homogeneous market (all high latency or all low latency) would not display, or greatly reduce, this anomaly.

5.2. Naïve orders

When the bid-ask spread is wide, LF traders tend to quote Naïve Orders. This is a misnomer as such orders are simply rational orders at their conception, that turn irrational (and therefore apparently naïve) when they hit the market, because the underlying prices have moved in the meantime. So a bid limit order at 102 (with reference to Fig. 1) was a perfectly rational quote when the order was launched but it turned to be an obsolete price when it eventually hit the book, because the price has moved downward in between conception and quotation time. In this case the percentage of quotes is significantly greater than the case of LFT only for all ratios up to 80%. For higher HF-to-LF ratios (i.e. 90% and 99%) the phenomenon is not noticeable as the low quoting rate of LF traders is not sufficient to generate such events. Interestingly, the Naïve Order phenomenon is more pronounced in quiet than in stressed markets.

5.3. Delta price

The highest-minus-lowest price (delta price) is significantly different from the LFT-only case at all HF-to-LF ratios except 99% in quiet markets. It is high for low ratios and then declines. It means that as long as there is some LFT activity the delay suffered by the slower trader causes them to execute market orders at worse prices than intended (as seen in the case of Bad Deals), pushing prices even more in the trending direction. As it was expected, trends exacerbate the phenomenon, displaying higher volatility and leading to a significant difference to the 0% ratio (LFT-only) even at 99% ratio.

5.4. Global volatility

Global Volatility is computed as the standard deviation of price movements and it is correlated to the Bad Deals observation, since in presence of high volatility, Low-Frequency traders tend to strike a higher number of Bad Deals, so further increasing volatility. As in case of Delta Price, a trending market amplifies the phenomenon.

5.5. Local volatility

Local Volatility is computed as the distance between the current price and the previous one, giving rise to a form of volatility at very short timescale, displaying the frequency of price change. Not surprisingly, this kind of volatility is not affected by market conditions: since it is computed within a very short time-window, the behaviour of prices over timescales just longer than the time-window itself are not relevant. Although the HF-to-LF ratio is significant again, except at 99%, results for quiet market do not significantly differ from those under stress.

Overall, the results of the simulation suggest that Bad Deals increase Global Volatility and Naïve Orders increase Local Volatility. The latter may only be noticed with sub-second observations because of its tendency to disappear quickly, but it is no less damaging for trading of slower investors, and if repeated over periods of time, to market stability.

6. Audit trail data

So far the theory suggests that. in presence of wide bid-ask spread, even moderate HFT activity is likely to produce phenomenon of Naïve Orders that, on its turn, amplify volatility. Unfortunately, the Bad Deal effect cannot be observed directly. A Bad Deal occurs when a slow trader wishes to execute a trade against a certain ('intended') price but by the time its order reaches the exchange the price has stepped back, resulting in an execution price worse than originally intended. Since the information about 'intended' price is not available in commercial data, the only way to estimate occurrence of Bad Deals is indirect, namely by observing Naïve Orders. The rationale behind this choice is that Naïve Orders and Bad Deals are generated by the same circumstances: delays suffered by LF traders with respect to HF ones. The difference between the two is that Naïve Orders are the consequence of delayed limit orders whereas Bad Deals are caused by delayed market orders. It therefore makes sense to consider the Naïve Orders to be proxies of Bad Deals. In order to verify whether the contemporaneous presence of large spread and HFT activity generate Naïve Orders, it is necessary to select an environment in which both certainly appeared. Audit trail data taken from any market over the last ten years (and likely more) will certainly display a high percentage of HFT activity. Speed-based strategies have been indeed quite common over the last decade: CFTC-SEC (2010) estimates HFT volumes in the equity market to be often in the order of 50% or more of total trading volume and even more recent studies (Serbera and Paumard, 2016) acknowledge a strong presence of HFT on financial markets. In the last decade there have been several episodes of wide bid-ask spread and since they are often associated with high volatility, it may be tempting to conclude that selecting the former does implicitly also selects the latter. Yet, the purpose here is to verify the presence of Naïve Orders associated to wide spread, and volatility only as its consequence. Since the Naïve Orders phenomenon does increase volatility at local level (the Local Volatility in Table 1), abnormal numbers of such orders would not indicate a role in generating stressed market conditions at global level (Global Volatility). The period selected for investigating the causal correlation between Naïve Orders and volatility is the six minutes that preceded and led to the Flash Crash on May 6, 2010. The choice might seem a bit too extreme: at the end of the day the Flash Crash was by no means an ordinary event. However, that day, and in particular that period of time, makes it an ideal field of study because of the wide spreads that were observed and since a phenomenon like Naïve Orders only acts at very short timescales, it cannot in any way be affected by Global Volatility on the same market or by the behaviour of other markets. On that day some securities were affected by stub quote phenomenon: extremely low bid (or high ask) limit order prices that were hit nonetheless. At 14:47:54 EDT Accenture (ticker = ACN) was traded at as low as $0.01 (from about $38 a few minutes earlier) and at 14:57:08 Sotheby (ticker = BID) enjoyed a spike from $33 up to $99,999. In order to avoid any bias, the market environment selected must be one which did not show such largely anomalous trades. Moreover, as many studies (Foresight, 2012; Durden, 2013; Johnson et al., 2013) report, the phenomenon of flash crashes seems pretty common and therefore taking one of them as reference makes good sense. A market that fits all the requirements is the E-mini S&P 500 futures market, in particular the contracts relative to June 2010, a close enough date to make investors particularly sensible to price movements. The data relative to May 6 shall be compared with data from nearby days to ensure that the wide bid-ask spread (particularly evident on May 6) was a definitive factor of high volatility – something that did not happen on other days. For correct comparison purposes, the same number of market events needs to be analysed, despite that the time ranges that accommodate those events are very different: whereas on May 6th the 580,684 events under observation occurred in less than six and a half minutes, on May 3rd they spanned nearly two hours, somehow less than one hour on May 4th and May 5th, and more than thirty-five minutes on May 7th (Table 2).

As shown in the example of Fig. 1, with a bid-ask spread of 100–105, a competitive but still rational limit order might be a bid at 101 or an ask at 104. Any limit order (either bid or ask) at 102 or 103 can be considered naïve, since the same competitive position in the book could be obtained at a better price. Analysis of the order books for the periods investigated above on May 3rd through 7th, 2010, reveals that on the day of the Flash Crash (May 6th) there were 431 so-called Naïve Orders, against 5 on May 3rd and 4th, and 16 on each of the 5th and 7th. Table 3 displays the Naïve Limit Orders quoted on May 6 between 18:39:00 and 18:45:28 GMT, where the first column shows the number of orders in the burst, the second column indicates the time at which the first order of the burst arrived at the exchange server, and the third column shows the price gap (a positive gap indicates a bid order, that is, a Naïve Limit

Table 2
Time windows for comparison of Naïve Orders spanning equal number of events.

Date	03-May	04-May	05-May	06-May	07-May
Time window	173318.954-192748.770	181107.807-191047.481	183348.932-192318.863	183900.007-184528.115	182906.396-190434.858
Duration	01:54:30	00:59:40	00:49:30	00:06:28	00:35:28
Events	580,864	580,864	580,864	580,864	580,865

Table 3
Naïve Orders on May 6, 2010.

No. of orders	Time	Max gap	No. of orders	Time	Max gap	No. of orders	Time	Max gap	No. of orders	Time	Max gap
1	183921825	0.5	1	184437754	−0.5	2	184511597	1.75	1	184519639	0.75
1	183934242	−0.5	1	184439203	0.5	1	184511716	−0.5	1	184519663	−0.5
2	184000955	0.5	1	184439230	−0.5	1	184511746	0.5	5	184519688	1
2	184025811	−0.5	1	184440942	0.5	1	184511748	−1.75	2	184519702	−0.5
1	184036191	0.5	1	184442992	−0.5	7	184511751	1.75	5	184519851	1
2	184132164	−0.5	1	184442995	0.5	1	184512185	−0.5	1	184520946	−0.5
3	184159458	0.75	1	184443577	−0.5	10	184512466	2	2	184521103	0.5
4	184244102	−1	1	184443586	0.5	1	184512491	−1.25	3	184521120	−0.5
1	184248306	0.75	1	184444065	−0.5	3	184512494	1.25	14	184521418	1.25
3	184250483	−0.5	2	184444431	0.5	1	184512500	−0.5	2	184522174	−0.5
1	184307638	0.5	2	184444890	−0.5	1	184512514	1.5	1	184522311	1.25
1	184308931	−0.5	4	184447676	0.75	1	184512516	−0.5	5	184522509	−2.25
1	184311106	0.75	1	184448484	−0.75	3	184512523	1.5	1	184522782	0.5
1	184311106	−0.5	7	184448493	1	1	184512531	−0.75	2	184522797	−0.75
1	184312358	0.5	4	184448569	−0.5	6	184512549	2.25	1	184522830	0.5
1	184319126	−0.5	2	184449377	0.5	3	184512573	−0.75	2	184522834	−0.5
1	184319127	0.5	1	184450209	−0.5	4	184512868	2.5	2	184522852	0.5
2	184321515	−1.25	3	184450737	0.75	5	184512965	−1	2	184523414	−0.5
3	184322074	0.5	7	184454087	−1	11	184512974	1.75	1	184523669	0.5
1	184322954	−0.5	1	184456025	0.5	2	184513580	−1	1	184523897	−0.5
1	184329606	0.5	3	184456105	−0.75	1	184513584	0.75	2	184524039	0.75
1	184332611	−0.5	7	184457779	1.25	1	184513585	−0.5	3	184524138	−0.5
1	184337250	0.5	2	184501782	−0.75	7	184513612	1.75	1	184525770	0.5
2	184337260	−0.75	2	184503005	0.5	1	184513887	−0.5	2	184525998	−0.5
1	184340130	0.5	1	184503187	−0.5	10	184514550	2	1	184526229	0.5
2	184340863	−0.75	2	184503197	0.75	5	184517974	−1.25	1	184526235	−0.5
2	184341150	0.75	2	184503389	−0.75	1	184517980	0.5	3	184526301	0.5
2	184343487	−1.25	1	184503411	1	2	184517993	−0.5	6	184526525	−3.25
1	184343613	0.5	3	184503412	−1	1	184518585	0.5	1	184526898	2.5
3	184345146	−0.75	3	184503415	0.5	1	184518609	−0.5	1	184526902	−1
1	184349267	0.5	2	184506230	−0.5	3	184518707	4.75	2	184526905	2.25
1	184353775	−0.5	4	184506565	0.75	2	184518708	−1	2	184526913	−0.5
1	184353777	0.5	1	184507202	−0.5	1	184518710	1.5	1	184526950	1
3	184402494	−0.75	1	184508191	0.5	5	184518710	−1.75	1	184526972	−1
4	184406276	0.75	3	184508324	−1	1	184518715	0.75	1	184526987	1.5
1	184417516	−0.5	5	184508593	0.75	3	184518715	−1.25	3	184526992	−1.5
1	184427912	0.5	2	184509769	−0.75	3	184518724	1.5	1	184527016	1.75
1	184428061	−0.5	1	184509776	0.5	1	184518727	−1.5	1	184527016	−1
2	184428072	0.75	5	184510033	−1	5	184518728	1	3	184527017	1.5
1	184428077	−0.5	1	184510254	0.5	1	184518734	−0.5	1	184527018	−0.5
1	184428078	0.5	2	184510260	−0.5	4	184518738	0.75	2	184527019	1.25
1	184428086	−0.5	2	184510982	1	2	184518764	−0.5	4	184527020	−1
1	184428125	0.5	1	184510986	−0.5	1	184518771	0.5	1	184527043	0.5
4	184428180	−0.5	9	184511003	0.75	1	184518859	−0.5	4	184527065	−0.5
2	184434818	0.5	1	184511585	−1.5	1	184518867	0.75	1	184527325	0.5
3	184435328	−0.5	2	184511588	0.75	2	184518872	−0.5	1	184527960	−0.5
1	184437745	0.5	1	184511589	−0.5	1	184518898	0.5	1	184527997	1.75
						2	184519220	−0.5	2	184527998	−3.25

Order whose price is higher than the second best order, and a negative one indicates an ask order with a price lower than the second best). Moreover, whereas on the other days the maximum gap between the top-of-book and the Naïve Orders was never larger than two ticks in either direction, on the wide spread day (May 6) the maximum gap for the bid book was 4.75 index points (at 18:45:18.707 GMT), equivalent to 19 ticks, and 3.25 index points on the ask book (at 18:45:27.998 GMT), equivalent to 13 ticks (the tick on the E-mini market is 0.25 index points). This suggests a very volatile and fast market when LF traders experienced a high delay if compared to HF traders. This table is informative as it shows evidence of several anomalies occurring on the day characterised by wide spread. A limit order posted with a gap of several ticks may be indicative of very high rate of price changes per unit of time (that is, high Local Volatility), so that in the period between the price observation and the quoting of a Naïve Order, the market has moved up, or down, a lot. A Naïve Limit Order is prone to be picked off by a trader sufficiently fast to exploit the opportunity, whereas a Naïve Market Order is likely to move the market abruptly, even without the conscious and explicit consent of the trader posting it. Commercially available data miss the information to decide about the reason behind the naivety, but the Naïve Order ratio between May 6 and the other days of the same week (86.2 on the first two days and 26.9 on the other two) suggests either a concentration of naïve traders on that day, or a frenetic HFT activity that caused slow traders to often accumulate large delays.

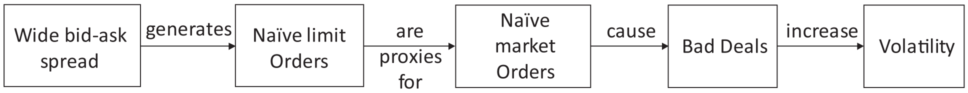

Fig. 3. Cause-effect relationship from Wide Bid-Ask Spread to Volatility.

7. Discussion

The set of mathematical equations presented in section3 and the simulation described in section 4, whose results were displayed in section 5, set the theoretical framework by stating that the contemporaneous presence of wide spread and HFT activity increases volatility. This happens because of the delays experienced by LF traders with respect to the faster participants, HF traders, lead them to generate both limit and market orders at a price level that may turn out obsolete when their orders eventually hit the books. The simulation finds a statistically significant increase in both Naïve Orders and Bad Deals. It must be clear that the number of Naïve Limit Orders in itself carries significance only for Local Volatility but it is a proxy of the number of Naïve Market Order, which likely contributed to abnormally increase market volatility but are not observable in a direct manner. Since with wide spread the number of Naïve Limit Orders per time unit is high and it can be considered a proxy for the number of Naïve Market Orders, the results of the simulation provide suggestive evidence to conclude that the number of Bad Deals was also high, and that this phenomenon does also increase volatility. As far as limit orders are concerned, analysis of audit trail data confirmed the expectations of the theory: wide bid-ask spread leads to an abnormal increase of Naïve Limit Orders. A sell market order targets a bid limit order priced at, say, 102. If it only finds the top of bid book at 100 when it actually hits the market, the trade will execute at that price (selling at a price lower than targeted), it will consume liquidity and, if frequently repeated, this process will increase volatility. The chain of logical steps is shown in Fig. 3.

The results of the simulation demonstrate that wide spread together with HFT activity generates an abnormal number of Naïve Limit Orders, which suggest also an abnormal number of Naïve Market Orders. Indeed, there is no reason to believe that the former does not also imply the latter as they are generated by the same phenomenon (LFT latency), the only difference being the selection of the order type (limit or market). The simulation also shows that a high number of Naïve Market Orders causes a large number of Bad Deals. These are market order that execute against the top of the book, whatever price they find, including a price worse or much worse than the price that was intended at the time when the market order was generated. Hitting the book at a worse than intended price increases volatility, because that leads to executing an order which exacerbates market movements, and potentially leads to extreme events, like flash crashes. All the results of the simulation vanish or are not significant in the LF-only scenario, show peaks and very high statistical significance for lower or medium HF-to-LF ratios and display declining, but still significant, numbers for higher ratios. The explanation of this occurrence is that when the market activity is of homogeneous type (all LFT or near all HFT) the ultra-fast traders do not have chances to exploit their speed advantage against slower traders because there are only a few, and therefore all the features found by the simulation are less pronounced. The most profitable situation seems to be when a low number of HF traders enjoy their advantage against a large number of slower counterparts, with little HF competition. When HFT activity grows and even more when it takes the lion share of the market activity, there will be few slow traders quoting Naïve Orders or striking Bad Deals, so restoring more stable market conditions. In a theoretical all-HFT market there is no reason to expect any speed disadvantage by any trader, as they will all act at ultra-fast speed.

8. Conclusion

This research studies a framework in which wide bid-ask spread is caused by exogenous reasons. It first presents a theoretical scenario described by a set of PDEs and by an Agent-Based Model implemented via a computer simulation and then compares the results with audit trail data analysis. The results of the simulation were consistent with the mathematical model and with expectations related to the increase of HFT activity: with LFT-only activity, no market instability was found to be caused by HFT: this is a trivial result and represents the base case used for comparison. When the level of HFT activity rises, market instabilities do appear, namely high volatility and abnormal behaviours, like high percentages of Naïve Orders and Bad Deals, which are on their turn a source of volatility. Local Volatility, a phenomenon likely to penalise slower traders also appears. As the level of HFT activity increases further, LF activity lowers with respect to total activity (LF + HF) and all the phenomena found with lower HF-to-LF ratios tend to diminish, because such phenomena were mostly caused by the obsolete information upon which LF orders were based. Stressed market conditions exacerbate the behaviours described earlier without substantially changing their shapes. The findings of this research suggest that, until HF traders will not be the large majority of market participants, their activity will generate the Naïve Orders and Bad Deals phenomena, the consequence being an increase in both local and global volatility, penalising slower traders and potentially destabilising the market.

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